Transition in a Plane Channel Flow in the Presence of a Magnetic Field.

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Abstract :

We investigate the linear evolution of perturbations in a magnetohydrodynamic channel flow with electrically insulating channel walls and a spanwise magnetic field. Linear optimal perturbation are computed. When the Hartmann number increases, dominant optimal perturbations are no more the classical streamwise modes. Performing direct numerical simulations using as initial condition optimal modes plus three-dimensional noise, we investigate how the transition to turbulence is affected by the magnetic field.

Résumé :

On étudie l'écoulement de Poiseuille plan d'un fluide conducteur soumis un champ magnétique orthogonal écoulement principal mais parallèle aux parois isolantes du canal. L' étude linéaire est fondée sur l'analyse des perturbations optimales conduisant aux plus fortes amplifications transitoires. À mesure que l'amplitude du champ magnétique augmente, on passe de modes non-normaux de type vortex longitudinaux, vers des modes obliques puis vers des modes bi-dimensionnels de type Orr. La Transition vers la turbulence est ensuite étudié par simulation numérique directe. On considère différentes valeurs du nombre de Hartmann Ha et on initie la simulation avec le mode non-normal prépondérant associée à cette valeur.

Key-words :

Optimal Perturbations ; Transition ; MHD

1 Introduction

We consider pressure-driven flow of an incompressible, electrically conducting fluid in an infinite plane channel between insulating walls with a magnetic field B in the spanwise direction (see Fig. 1). Such flows can be found in numerous metallurgical and materials processing applications, e.g. in electromagnetic flow control, in continuous steel casting and in growth of large silicon crystals. Another area of applications is the liquid metal (Li or Pb-17Li) cooling blankets of breeder type for fusion reactors. Our numerical study is performed within the quasi-static approximation, whereby the governing equations reduce to the Navier-Stokes system with the additional Lorentz force (Davidson (2001)). The length and velocity scales are the laminar centerline velocity U and the channel half width a. The non-dimensional basic velocity profile is the parabolic Poiseuille profile. Nondimensional parameters are the Reynolds number $Re = Ua/\nu$ and the Hartmann number $Ha = aB\sqrt{\sigma/\rho\nu}$, where σ denotes the electric conductivity and B the magnetic field strength.

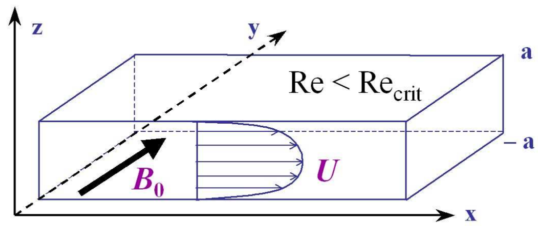

Figure 1: Sketch of the Problem.

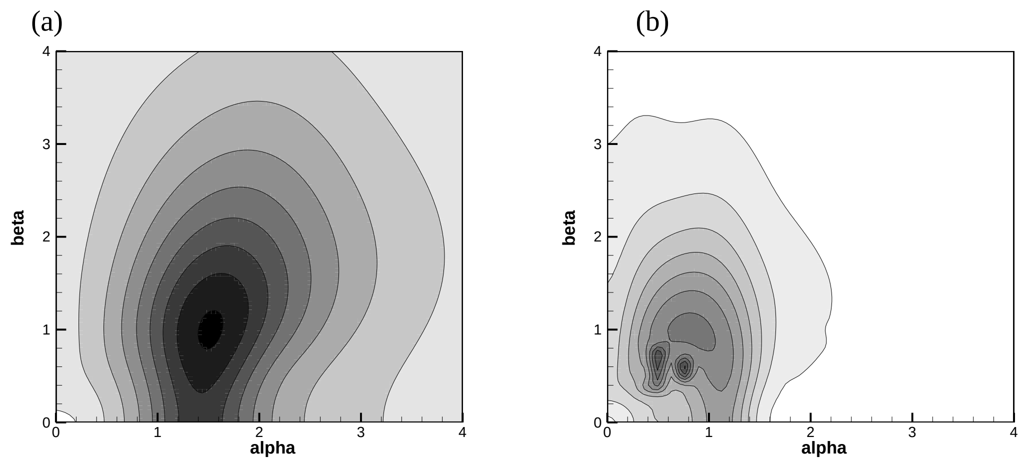

Figure 2: Isolevels of energy amplification $\hat{G}(\alpha,\beta)$ for $Ha = 50$ at different moments in time T. (a) Global maximum at $T = 15$, (b) three local peaks at $T \approx 28$.

Note that recent stability works on channel flows in the presence of magnetic field have been performed in an MHD context from an experimental viewpoint (Moresco *et al*. (2004)) or a theoretical one (Airiau *et al*. (2004)). In this last paper, the authors were considering non-normal modes but with a magnetic field orthogonal to the walls.
We perform a systematic study of the optimal linear perturbations providing the strongest transient growth for different Ha at a fixed, subcritical Reynolds number $Re = 5000$. Among the possible transition scenarios, we then focus on the one based on the nonlinear transient growth of these optimal perturbations and their subsequent three-dimensional breakdown.

2 Transient growth of linear perturbations

In the linear analysis, we study the evolution of decoupled monochromatic Fourier modes with the wavenumbers α and β in the streamwise and spanwise directions. The flow is linearly stable, i.e. all eigensolutions decay exponentially. However, linear combinations of eigenmodes can experience substantial transient algebraic growth before they eventually decay. To quantify the amplification at time T we use the kinetic energy E of the perturbations, whereby the individual contributions of each wavenumber pair (α,β) can be considered independently. The amplification gain of any given mode at time T is the ratio $E(T)/E(0)$. This ratio is maximized over all possible initial vertical shapes by an optimization procedure (Schmid (2001)) to give the maximum amplification $\hat{G}(T,\alpha,\beta)$ of the disturbances with the wavenumbers (α,β) at the time T.
For $Ha = 0$, streamwise vortices with $\alpha = 0$ provide the largest transient amplification. These modes are strongly damped by the magnetic field, and are therefore supplanted by oblique

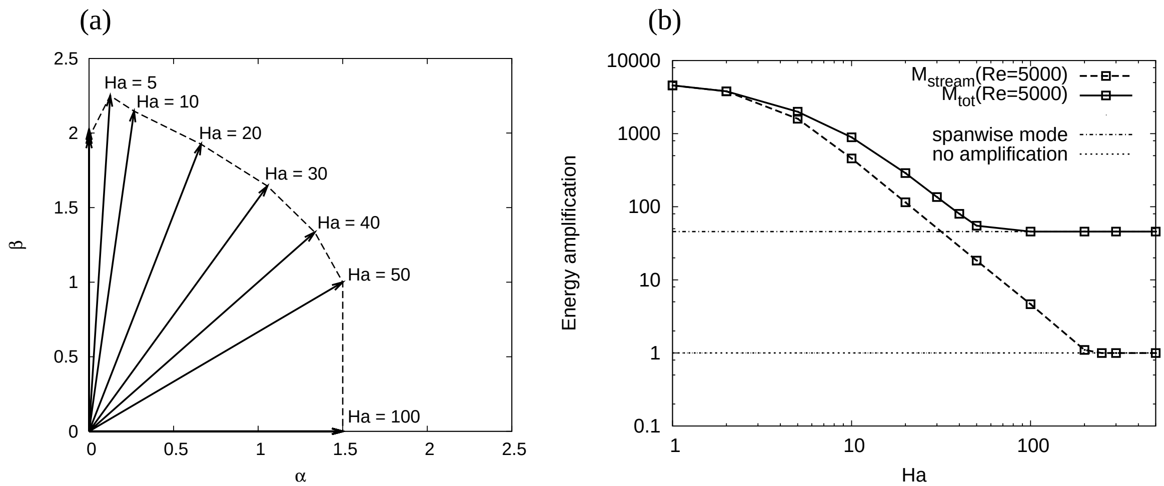

Figure 3: Optimal wavevector vs. Ha (a) and amplification gains $\hat{M}_{tot}$ and $\hat{M}_{stream}$ for global and streamwise optimal modes (b).

modes with $\alpha > 0$ for $Ha \geq 5$. Fig. 2 shows isolines of $\hat{G}$ for $Ha = 50$ for different times T. The contours indicate variation of $\hat{G}$ from low (white regions) to high (black regions) values. The highest amplification occurs for $T \approx 15$ (Fig. 2a), but there may be several co-existing local maxima, e.g. at $T \approx 28$ (Fig. 2b). Maximization with respect to T, α and β provides the maximum amplification $\hat{M}$ and the corresponding wavevector (α, β) as functions of Ha, which are shown in Fig. 3. The transient growth of oblique perturbations is also reduced for $Ha > 0$, and the oblique angle of the optimal modes increases monotonically with Ha. For $Ha \geq 100$, the spanwise Orr-mechanism modes unaffected by the magnetic field become the modes with strongest transient amplification. In Fig. 3(b), these modes provide the constant amplification level for $Ha \geq 100$. We also show the maximum amplification of streamwise rolls with $\alpha = 0$ for comparison. For these modes, the amplification $\hat{M}_{stream}$ eventually reduces to unity, i.e. they experience no transient growth for sufficiently large Ha. Remarkably, we find scaling relations $\beta \sim Ha^{-1}$ and $\hat{M}_{stream} \sim Ha^{-2}$ for these modes.

3 Direct simulations of transition

At a subcritical Reynolds number, transient growth of initial perturbations may result in a finite-amplitude modification of the basic velocity profile that may be sufficiently strong so that the flow becomes unstable to three-dimensional noise. This approach corresponds to the so-called two-step scenario, which was shown to be pertinent for other parallel shear flows, such as the plane Poiseuille (Reddy *et al.* (1998)), pipe Poiseuille (Zikanov (1996)), and Hartmann (Krasnov *et al.* (2004)) flows.

In this context, we study the nonlinear evolution and transition of the basic flow initially modulated by an optimal linear mode of a specified amplitude. The representative flow regimes with Ha ranging between 10 and 100 are considered. The initial kinetic energy of the perturbations $E(0)$ varies between 10^{-5} and 10^{-2} relative to that of the basic flow. Weak three-dimensional noise with energy $E_{3D} = 0.01E(0)$ is added at the moment of maximum linear amplification.

For the classical case $Ha = 0$, the linear analysis gives the maximum linear amplification $M_{tot}(Re = 5000, 0) \approx 4897$ for purely streamwise vortices with $(\alpha_{opt}, \beta_{opt}) = (0, 2.04)$ The nonlinear evolution of the streamwise modes indicates that a perturbation energy of streamwise vortices of amplitude $E(0) = 10^{-5}$ with a noise energy $E_{3D} = 0.01E(0)$ is sufficient to trigger

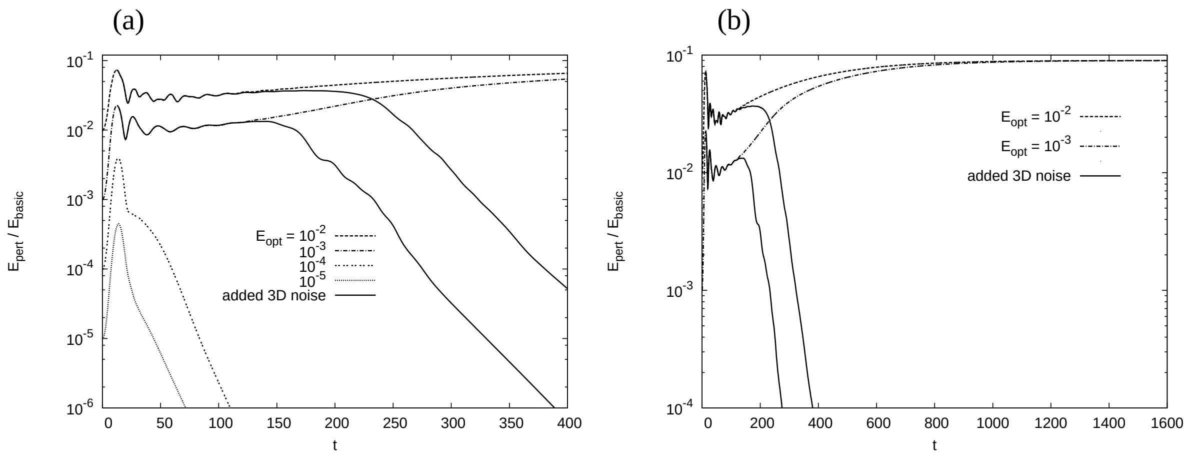

Figure 4: The nonlinear evolution of perturbation energy $E(t)$ starting with Orr mode at $Re = 5000$ and different Ha. (a) $Ha = 100$, the early stage ($t \leq 400$) with initial amplitude $E(0) = 10^{-5}$ ... 10^{-2} and $3D$ noise of amplitude $E_{3D} = 10^{-2}E(0)$ added at $t = T_{opt}$ for the bold curves. (b) Case $Ha = 100$, the complete evolution ($t \leq 1600$) up to a sustained $2D$ finite amplitude state for $E(0) = 10^{-2}$ and 10^{-3}. $3D$ noise of amplitude $E_{3D} = 10^{-2}E(0)$ is added at $t = T_{opt}$ for the bold curves.

the transition.

For the modest Hartmann number $Ha = 10$, there is already a significant effect of the magnetic field upon the streamwise vortices since they do not induce transition to turbulence if $E(0) < 10^{-4}$. The maximum energy amplification is now provided by an oblique mode. For such an optimal mode, even amplitude $E(0) = 10^{-5}$ is sufficient to trigger the transition.

The oblique orientation of the optimal modes at moderate Ha opens an interesting possibility of simultaneous growth of two superimposed symmetric modes (α, β) and $(\alpha, -\beta)$. We found that these modes efficiently serve as secondary disturbances to each other so there is no need for adding noise to trigger the transition. Non-linear interaction between the symmetric modes generates other modes experiencing transient growth. In particular, the spanwise mode $(2\alpha, 0)$ was shown to provide approximately 10% increase of the total energy amplification of the perturbations.

For strong magnetic fields at Ha larger than 30, no transition to turbulence was detected. We have checked this for all families of optimal linear modes, each providing the maximum amplification in a certain time range and all flow excitations (single mode plus 3D noise, superposition of symmetric oblique modes and, additionally, superposition of symmetric oblique modes + 3D noise). The nonlinear effects become noticeable when the initial perturbation energy becomes $E(0) \geq 10^{-3}$ but the transition to turbulence is never triggered.

For even stronger magnetic fields (Ha larger than 100), only spanwise perturbations are experiencing appreciable transient growth. For sufficiently strong initial spanwise perturbations e.g. $E(0) \geq 10^{-3}$, the growth of such modes and their non-linear saturation lead to establishing a spanwise-independent secondary flow observed earlier in the two-dimensional simulations of (Jimenez (1990)). However this flow when perturbed by three-dimensional fluctuations, leads to a transition back into the base state (see figure 4). Further investigations are needed to resolve interesting related questions, such as that of stability of the two-dimensional solutions at even higher Ha and the role played by the spanwise Orr modes in the transition at supercritical Re.

4 Conclusions

The transient growth of linear perturbations in laminar channel flow is suppressed by a spanwise magnetic field, and the optimal perturbations change from streamwise rolls to oblique rolls. The tilting angle of the oblique optimal modes increases monotonically with the magnetic field strength, until only the spanwise Orr-mechanism vortices provide transient amplification.
The nonlinear transition is caused by the transient growth and subsequent breakdown of optimal perturbations, which take the form of one or two symmetric optimal modes (streamwise, oblique, or spanwise modes depending on Ha) with low-amplitude three-dimensional noise added at the moment of strongest energy amplification. Sufficiently strong magnetic field (Ha larger than approximately 30) is found to completely suppress the instability. At smaller Hartmann numbers, the transition is observed but it is strongly modified in comparison to the classical non-magnetic case.

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References

Airiau Ch. and Castets M. 2004 On the amplification of small disturbances in a channel flow with a normal magnetic field. *Phys.of Fluids* **16**, 8; 2991-3005

Davidson, P. A. 2001 An Introduction to Magnetohydrodynamics *Cambridge University Press*

Jimenez J. 1990 Transition to turbulence in two-dimensional Poiseuille flow *J. Fluid Mech.* **218**, 265-297

Krasnov D. S.et al 2004 Numerical study of the instability of the Hartmann layer. *J. Fluid Mech.* **504**, 183-211

Moresco R. and Alboussière Th. 2004 Experimental study of the instability of the Hartmann layer. *J. Fluid Mech.* **504**, 167–181

Reddy S. C. et al 1998 On the stability of stream-wise streaks and transition thresholds in plane channel flow. *J. Fluid Mech.* **365**, 269–303

Schmid, P. J. and Henningson D. S. 2001 Stability and Transition in Shear Flows *Springer, Berlin Heidelberg New York*

Zikanov, O. 1996 On the instability of pipe Poiseuille flow. *Phys.of Fluids* **8**, 2923-2932